Pergamon

Int. J. Heat Mass Transfer. Vol. 40, No. 16, pp. 3819–3826, 1997

Printed in Great Britain
0017–9310/97 $17.00+0.00

PII: S0017–9310(97)00038–0

Microscales of rotating turbulent flows

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(*Received* 13 *September* 1996 *and in final form* 19 *January* 1997)

Abstract—In terms of the usual Prandtl and Taylor numbers, a combined dimensionless number

$$\Pi_C \sim \frac{TaPr^2}{1+Pr}$$

is introduced for rotating thermal flows. Depending on this number and relative to an integral scale, a thermal boundary layer thickness

$$\frac{\delta_\theta}{\ell} \sim \Pi_C^{-1/4}$$

for the laminar case and a thermal Kolmogorov scale

$$\frac{\eta_\theta}{\ell} \sim \Pi_C^{-1/3}$$

for the turbulent case, are developed. In terms of these scales, a model for laminar heat transfer

$$Nu \sim \Pi_C^{1/4}$$

and another for turbulent heat transfer

$$Nu \sim \Pi_C^{1/3}$$

are proposed. Experimental literature is correlated by these models. © 1997 Elsevier Science Ltd.

1. INTRODUCTION

The Taylor–Couette flow between rotating cylinders is one of the most extensively studied problems of fluid mechanics because of its practical significance (for a recent review, see [1]). Linear and non-linear stability analyzes have been used to predict flow transitions [2, 3] while extensive experiment work is devoted to prediction of the torque relation [4, 5]. Batchelor [6] proposed an 'inviscid-core-and-boundary-layer' model for the torque relation in laminar flow. By using an analogy between Taylor–Couette and stratified flows [7], Smith and Townsend [8] proposed models for the torque relation in turbulent flow. Recent studies appear to concentrate on flow transitions and chaotic behavior [9]. Studies on heat transfer in rotating flows, however, are relatively sparse [10–13]. In addition, the existence and fundamental significance of Π_C for heat transfer in these flows have been apparently overlooked and is the motivation of this study.

2. A DIMENSIONLESS NUMBER

Dimensionless numbers characterizing buoyancy driven flows are the Rayleigh and Prandtl numbers, *Ra* and *Pr*, respectively. A dimensionless number, Π_N, recently proposed by Arpaci [14] goes one step further and describes these flows by a combination of *Ra* and *Pr*. More recently, Arpaci and Selamet [15] and Arpaci and Kao [16] demonstrated the fundamental significance of this number by its extensions to buoyancy driven flames and two-phase films, respectively.

Dimensionless numbers for centrifugally driven thermal flows are usually assumed to be the Taylor and Prandtl numbers, *Ta* and *Pr*. However, in view of the foregoing Arpaci and co-worker studies [14–21], a dimensionless number involving a combination of *Ta* and *Pr* is expected and to be developed below.

Let a centrifugally driven momentum balance be

$$F_C \sim F_I + F_V \tag{1}$$

where F_C, F_I and F_V, respectively, denoting centrifugal, inertial and viscous forces. Also, let the associated thermal energy balance be

$$Q_H \sim Q_K \tag{2}$$

where Q_H and Q_K, respectively, denoting enthalpy flow and conduction. Then, from equation (1)

NOMENCLATURE

A	amplitude
C	constant
C_T	torque coefficient
d	gap width
F	force
F_g	geometric factor
$G_{(\delta)C}$	Görtler number
h	heat transfer coefficient
k	heat conductivity
ℓ	integral length
Nu	Nusselt number
Pe_C	Peclet number for rotating flows
Pr	Prandtl number
Q	heat flux
R	radius
Ra	Rayleigh number
Re	Reynolds number
Rt	Rayleigh–Taylor number
S_{ij}	mean stress tensor
s_{ij}	stress tensor
Ta	Taylor number
t	integral time scale
U	steady mean velocity
u	integral velocity scale.

Greek symbols

α	thermal diffusivity
δ	boundary layer thickness
ε	dissipation
η	Kolmogorov length scale
Θ	steady mean temperature
θ	temperature
λ	Taylor scale
G	torque per unit length
G^*	critical torque per unit length
$\mathscr{D}$	diffusion
$\mathscr{K}$	kinetic energy
$\mathscr{P}$	production
μ	dynamic viscosity
ν	kinematic viscosity
Π_C	dimensionless number for rotating flows
$\mathscr{T}$	torque power
ω	angular velocity.

Subscripts

C	centrifugal
H	enthalpy
I	inertial
K	conduction
V	viscous
γ	centrifugal
θ	thermal.

Superscripts

$\tilde{}$	instantaneous
$\bar{}$	mean
B	Batchelor scales
C	Oboukhov–Corrsin scales.

$$\frac{F_C}{F_I+F_V} = \frac{F_C/F_V}{F_I/F_V+1} \tag{3}$$

and from equation (2)

$$Q_H/Q_K \tag{4}$$

the numeral 1 in equation (3) implying order of magnitude. Although the force ratios of equation (3) and the energy ratio of equation (4) are dimensionless, they are usually expressed in terms of velocity which is a dependent variable in torque driven flows:

$$\frac{F_C}{F_V} \sim \frac{\omega^2 R\ell^2}{\nu V}, \quad \frac{F_I}{F_V} \sim \frac{\rho V\ell}{\mu}, \quad \frac{Q_H}{Q_K} \sim \frac{\rho c V\ell}{k} \tag{5}$$

where ω, R and ℓ, respectively, being the angular frequency, centrifugal and geometric scales, while the rest of the notational being conventional. The only combination of equations (3) and (4) independent of velocity and appropriate for centrifugally driven flows is

$$\Pi_C \sim \frac{(F_C/F_V)Q_H/Q_K}{(F_I/F_V)Q_K/Q_H+1} \tag{6}$$

or

$$\Pi_C \sim \frac{Rt}{1+Pr^{-1}} \tag{7}$$

where

$$Rt = \frac{\rho\omega^2 R\ell^3}{\mu\alpha} \tag{8}$$

is the Raleigh–Taylor number which appears to be overlooked in the literature. Note that

$$Rt = TaPr \tag{9}$$

where

$$Ta = \frac{\omega^2 R\ell^3}{\nu^2} \tag{10}$$

is the Taylor number. Note further that the Taylor number is actually a Grashof number for centrifugally driven flows. In terms of Ta

$$\Pi_C \sim \frac{TaPr^2}{1+Pr} \tag{11}$$

and the two limits of equation (11) are

$$\lim_{Pr \to 0} \Pi_C \to RtPr = TaPr^2 = Pe_C \tag{12}$$

Pe_C being a Peclet number for these flows and

$$\lim_{Pr \to \infty} \Pi_C \to Rt = TaPr. \tag{13}$$

The next section on the centrifugally driven laminar flows proves convenient in the following section on the objective of the study which is the microscales of torque and heat transfer in turbulent flows depending on Π_C.

3. LAMINAR FLOW

On dimensional grounds, equations (1) and (2), respectively, lead to

$$F_C \sim \left(\frac{u^2}{\ell} + v\frac{u}{\delta^2}\right)\rho\ell^3 \tag{14}$$

and

$$u\frac{\theta}{\ell} \sim \alpha\frac{\theta}{\delta_\theta^2}. \tag{15}$$

To proceed further, following the Squire postulate [22] for buoyancy driven flows, assume

$$\delta \sim \delta_\theta \tag{16}$$

in equation (14). This is an often misinterpreted pivotal assumption. It postulates the secondary importance of the difference between δ and δ_θ for heat transfer rather than suggesting equality of these thicknesses. In view of equation (16), elimination of velocity between equations (14) and (15) leads to

$$\delta_\theta^4 \sim \ell^4\left(1+\frac{\alpha}{v}\right)\frac{\mu\alpha}{F_C} \tag{17}$$

which may be reduced, in terms of $F_C = \rho\omega^2 R\ell^3$, to

$$\frac{\delta_\theta}{\ell} \sim \Pi_C^{-1/4}. \tag{18}$$

For isothermal flow, dimensional equipartition of centrifugal force gives

$$F_C \sim \rho u^2\ell^2 \sim \mu\frac{u}{\delta^2}\ell^3 \tag{19}$$

or

$$u \sim \left(\frac{F_C}{\rho\ell^2}\right)^{1/2} \sim \delta^2\left(\frac{F_C}{\mu\ell^3}\right) \tag{20}$$

and

$$\frac{\delta}{\ell} \sim Ta^{-1/4}. \tag{21}$$

4. TURBULENT FLOW

Following the usual practice, let the instantaneous velocity and temperature of a rotating turbulent flow be decomposed into a temporal mean (denoted by capital letters) and fluctuations

$$\tilde{u}_i = U_i + u_i \quad \text{and} \quad \tilde{\theta} = \Theta + \theta$$

and assume U_i and Θ be statistically steady. Then, the balance of the mean kinetic energy of velocity fluctuations

$$\mathscr{K} = \tfrac{1}{2}\overline{u_i u_i}$$

yields (for example, see [23])

$$U_j\frac{\partial \mathscr{K}}{\partial x_j} = -\frac{\partial \mathscr{D}_j}{\partial x_j} - \mathscr{P}_\gamma + \mathscr{P} + \varepsilon \tag{22}$$

where

$$\mathscr{D}_j = \overline{\left(\frac{p}{\rho} + \frac{1}{2}u_i u_i\right)u_j} - 2v\overline{u_i s_{ij}}$$

is the diffusion

$$\mathscr{P}_\gamma = \overline{u_i u_j}\frac{\partial U_i}{\partial x_j} \tag{23}$$

is the centrifugal production*

$$\mathscr{P} = -\overline{u_i u_j}S_{ij} \tag{24}$$

is the inertial production and

$$\varepsilon = -2v\overline{s_{ij}s_{ij}} \tag{25}$$

is the viscous dissipation of turbulent energy.

Also, the balance of the root mean square of temperature fluctuations

$$\mathscr{K}_\theta = \frac{1}{2}\overline{\theta^2}$$

gives

$$U_j\frac{\partial}{\partial x_j}(\mathscr{K}_\theta) = -\frac{\partial}{\partial x_j}(\mathscr{D}_\theta)_j + \mathscr{P}_\theta - \varepsilon_\theta \tag{26}$$

where

$$(\mathscr{D}_\theta)_j = \frac{1}{2}\overline{\theta^2 u_j} - \alpha\frac{\partial}{\partial x_j}\left(\frac{1}{2}\overline{\theta^2}\right)$$

is the thermal diffusion

$$\mathscr{P}_\theta = -\overline{u_j\theta}\frac{\partial \Theta}{\partial x_j} \tag{27}$$

is the thermal production and

* In cylindrical coordinates, this production becomes

$$\mathscr{P}_\gamma = \frac{\overline{uw}}{r}\frac{\mathrm{d}}{\mathrm{d}r}(Ur)$$

u being in the direction of rotation and w in the direction of shear.

$$\varepsilon_\theta = \alpha \frac{\overline{\partial\theta}\,\overline{\partial\theta}}{\partial x_j \partial x_j} \quad (28)$$

is the thermal dissipation.

For a homogeneous pure shear flow (in which all averaged quantities except U_i and Θ are independent of position and in which S_{ij} and $\partial\Theta/\partial x_j$ are constant), equations (22) and (26) reduce to

$$\mathscr{P}_\gamma = \mathscr{P} + \varepsilon \quad (29)$$

and

$$\mathscr{P}_\theta = \varepsilon_\theta. \quad (30)$$

Equation (29) states that the centrifugal production is partly converted into inertial production and partly into viscous dissipation. On dimensional grounds, assuming $S_{ij} \sim u/\ell$ and $\partial\Theta/\partial x_j \sim \theta/\ell$, equations (29) and (30) may be written as

$$\mathscr{P}_\gamma \sim \frac{u^3}{\ell} + \nu \frac{u^2}{\lambda^2} \quad (31)$$

and

$$u\frac{\theta^2}{\ell} \sim \alpha \frac{\theta^2}{\lambda_\theta^2} \quad (32)$$

where u and θ, respectively, denote the rms values of velocity and temperature fluctuations, ℓ is an integral scale, λ and λ_θ are the Taylor scales. Equations (31) and (32) imply isotropic mechanical and thermal dissipations. Note that the isotropic dissipation is usually a good approximation for any turbulent flow [23].

To proceed further, assuming the Squire postulate to be independent of flow conditions and following equation (16), let

$$\lambda \sim \lambda_\theta. \quad (33)$$

Then, elimination of velocity between equations (31) and (32) results in a thermal Taylor scale

$$\lambda_\theta \sim \ell^{1/3}\left(1+\frac{1}{Pr}\right)^{1/6}\left(\frac{\nu\alpha^2}{\mathscr{P}_\gamma}\right)^{1/6}, \quad Pr \geqslant 1 \quad (34)$$

which may be rearranged as

$$\lambda_\theta \sim \ell^{1/3}(1-Pr)^{1/6}\left(\frac{\alpha^3}{\mathscr{P}_\gamma}\right)^{1/6} \quad Pr \leqslant 1 \quad (35)$$

where equation (34) explicitly includes the limit for $Pr \to \infty$ and is convenient for fluids with $Pr \geqslant 1$, and equation (35) explicitly includes the limit for $Pr \to 0$ and is convenient for fluids with $Pr \leqslant 1$.

Now, consider the local (isotropic) behavior of the homogeneous flow (in a sublayer next to a wall or in a vortex tube) and let λ_θ, ℓ (which are no longer distinguished) be replaced by an asymptotic scale

$$\begin{pmatrix}\ell \\ \lambda_\theta\end{pmatrix} \to \eta_\theta. \quad (36)$$

Then equations (34) and (35) are, respectively, reduced to a thermal Kolmogorov scale for centrifugally driven flows*

$$\eta_\theta \sim \left(1+\frac{1}{Pr}\right)^{1/4}\left(\frac{\nu\alpha^2}{\mathscr{P}_\gamma}\right)^{1/4}, \quad Pr \geqslant 1 \quad (37)$$

$$\eta_\theta \sim (1+Pr)^{1/4}\left(\frac{\alpha^3}{\mathscr{P}_\gamma}\right)^{1/4}, \quad Pr \leqslant 1. \quad (38)$$

Now, it is a simple matter to show that

$$\lim_{Pr\to\infty} \eta_\theta \to \left(\frac{\nu\alpha^2}{\mathscr{P}_\gamma}\right)^{1/4} \quad (39)$$

and

$$\lim_{Pr\to\infty} \mathscr{P} \to 0 \quad (40)$$

which implies, in view of equation (29)

$$\mathscr{P}_\gamma \sim \varepsilon. \quad (41)$$

Then, from equations (39) and (41),

$$\lim_{Pr\to\infty} \eta_\theta \to \eta_\theta^{\mathrm{B}} \sim \left(\frac{\nu\alpha^2}{\varepsilon}\right)^{1/4} \quad (42)$$

which is the Batchelor scale. Also

$$\lim_{Pr\to 0} \eta_\theta \to \left(\frac{\alpha^3}{\mathscr{P}_\gamma}\right)^{1/4} \quad (43)$$

$$\lim_{Pr\to 0} \varepsilon \to 0 \quad (44)$$

which implies, in view of equation (29)

$$\mathscr{P}_\gamma \sim \mathscr{P} \quad (45)$$

and, in a viscous layer order of magnitude thinner than η_θ

$$\mathscr{P} \to \varepsilon. \quad (46)$$

Then, matching the inner limit of equation (45) to the outer limit of equation (46) leads to equation (41) and equation (43) becomes

$$\lim_{Pr\to 0} \eta_\theta \to \eta_\theta^{\mathrm{C}} \sim \left(\frac{\alpha^3}{\varepsilon}\right)^{1/4} \quad (47)$$

which is the Oboukhov–Corrsin scale. Finally, for $Pr \to 1$, because of (order of magnitude) equipartition of the centrifugal production into inertial production and viscous dissipation, equation (29) becomes

$$\mathscr{P}_\gamma \sim 2\varepsilon \quad (48)$$

and equations (37) and (38) both lead to

* The first numeral 1 in the right-hand side of equations (34), (35), (37) and (38) is related to the numeral 1 of equation (3) and implies order of magnitude.

$$\lim_{Pr\to 1} \eta_\theta \to \eta \sim \left(\frac{\nu^3}{\varepsilon}\right)^{1/4} \quad (49)$$

which is the celebrated Kolmogorov scale.

The relation between the thermal microscales and the integral scale may now be obtained by eliminating the factor $(1+1/Pr)(\nu\alpha^2/\mathscr{P}_\gamma)$ between equations (34) and (37). This readily yields

$$\left(\frac{\eta_\theta}{\lambda_\theta}\right)^2 = \frac{\lambda_\theta}{\ell}. \quad (50)$$

Equations (35) and (38) lead to the same relation, as expected. The forgoing scales are utilized in the next section on the development of torque and heat transfer relations for rotating flows. Before this development, however, the relations between these scales and the dimensionless number Π_C (recall equation (7), need to be shown.

Note that $\mathscr{P}_\gamma$ depends on velocity, and equations (34) and (35), and (37) and (38) expressed in terms of velocity cannot be ultimate forms of the Taylor and Kolmogorov scales for rotating flows. To eliminate any velocity dependence, assume first

$$\mathscr{P}_\gamma \sim \mathscr{T} \quad (51)$$

where imposed torque power $\mathscr{T}$ in terms of the centrifugal force is

$$\mathscr{T} \sim \omega^2 R u \quad (52)$$

R being the radius of the cylinder and u the homogeneous velocity given by equation (32),

$$u \sim \alpha \frac{\ell}{\lambda_\theta^2}. \quad (53)$$

Then, for thermal flow, equations (34) and (35) lead to

$$\frac{\lambda_\theta}{\ell} \sim \Pi_C^{-1/4}, \quad (54)$$

and, for isothermal flow, to*

$$\frac{\lambda}{\ell} \sim Ta^{-1/4}. \quad (55)$$

The isotropic limit of equation (53) is

$$u \sim \frac{\alpha}{\eta_\theta}. \quad (56)$$

For thermal flow, equations (37) and (38) lead, in terms of equation (56), to

$$\frac{\eta_\theta}{\ell} \sim \Pi_C^{-1/3} \quad (57)$$

and, for isothermal flow, to

$$\frac{\eta}{\ell} \sim Ta^{-1/3}. \quad (58)$$

The next two sections are devoted to models for torque and heat transfer in terms of the foregoing scales.

5. TORQUE

5.1. *Laminar flow*

In terms of shear stress, the torque relation is

$$G \sim R^2 \mu \frac{\omega R}{\delta} \quad (59)$$

or

$$G \sim \frac{\mu\omega R^3}{\ell} \times \frac{\ell}{\delta} \quad (60)$$

which gives, in terms of equation (21)

$$G \sim G^* Ta^{1/4} \quad (61)$$

where

$$G^* = 2\pi\mu\omega R^3/\ell \quad (62)$$

is the torque relation for the Couette flow.

5.2. *Turbulent flow*

For the case of turbulent flow, equation (59) becomes

$$G \sim R^2 \mu \frac{\omega R}{\eta} \quad (63)$$

which may be rearranged as

$$G \sim \frac{\mu\omega R^3}{\ell} \times \frac{\ell}{\eta}. \quad (64)$$

Then, in terms of equation (58)

$$G \sim G^* Ta^{1/3} \quad (65)$$

where

$$G^* = 2\pi\mu\omega R^3/\ell \quad (66)$$

is the torque relation for the Couette flow. Equation (65) is identical, except for a constant, to the torque relation proposed by Smith and Townsend [8]

$$G = U_1^2 R_1^2 (2C)^{-4/3} \left(\frac{U_1 R_1}{\nu}\right)^{-1/3} \quad (67)$$

where $U_1 = \omega R_1$ and C is a numerical constant. Note that torque power $\mathscr{T}$, convenient for microscale devel-

* Note the similarity between equations (21) and (55) which apply for different flow conditions.

opment, is related to the usual definition of torque G by

$$\rho R^2 \mathscr{T} = \frac{dG}{dt}. \qquad (68)$$

6. HEAT TRANSFER

6.1. *Laminar flow*

On dimensional grounds, the balance between enthalpy flow and conduction given by equation (2) leads to

$$Nu \sim \frac{\ell}{\delta_\theta} \qquad (69)$$

or, in terms of equation (18), to

$$Nu \sim \Pi_C^{1/4}. \qquad (70)$$

Two limits of equation (29) for $Pr \to 0$ and $Pr \to \infty$ are, respectively,

$$Nu \sim (RtPr)^{1/4} = (TaPr^2)^{1/4}, \quad Pr \ll 1 \qquad (71)$$

and

$$Nu \sim (Rt)^{1/4} = (TaPr)^{1/4}, \quad Pr \gg 1. \qquad (72)$$

6.2. *Turbulent flow*

For the case of turbulent flow, equation (69) becomes

$$Nu \sim \frac{\ell}{\eta_\theta} \qquad (73)$$

or, in terms of equation (57), to

$$Nu \sim \Pi_C^{1/3}. \qquad (74)$$

Two limits of equation (29) for $Pr \to 0$ and $Pr \to \infty$ are, respectively,

$$Nu \sim (RtPr)^{1/3} = (TaPr^2)^{1/3}, \quad Pr \ll 1 \qquad (75)$$

and

$$Nu \sim (Rt)^{1/3} = (TaPr)^{1/3}, \quad Pr \gg 1. \qquad (76)$$

The present study generalizes, in terms of Π_C, a previous study by Arpaci and Kao [24] who considered the special cases given by equations (71), (72), (74) and (75).

7. COMPARISON WITH EXISTING DATA AND DISCUSSION

Dimensionless torque relations for the laminar and turbulent Taylor–Couette flows,

$$C_T \sim Ta^{1/4}, \quad C_T \sim Ta^{1/3} \qquad (77)$$

are well-known. Present study provides a microscale foundation for the turbulent case of these relations. Early experimental data obtained by Taylor [25], Wendt [26] and analyzed by Donnelly and Simon [4] are correlated by

$$C_T = 0.23 Ta^{1/4}, \quad C_T = 0.063 Ta^{1/3}. \qquad (78)$$

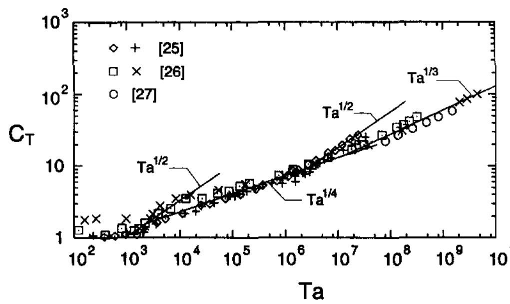

Fig. 1. Proposed torque relations with $0.23Ta^{1/4}$ for laminar flow and $0.063Ta^{1/3}$ for turbulent flow.

Figure 1 shows these correlations. There are also similar correlations. For example, using the analogy between Couette and stratified flows, Smith and Townsend [8] propose a torque relation which can be rearranged as

$$C_T = (2C)^{-4/3} Ta^{1/3}. \qquad (79)$$

Barcilon and Brindley [27] propose, for a Taylor problem and a boundary Görtler problem,

$$G = \frac{1}{2}\left(\frac{\Omega_1^2 R_1^4}{\nu^2}\right)^{1/3} G_{(\delta)c}^{-1/3}\left(\frac{d}{R_1}\right) \sim Ta^{1/3} \qquad (80)$$

where $G_{(\delta)c}$ is the Görtler number.

In addition to correlations given by equation (78), Fig. 1 shows two short transition regimes correlating with $Ta^{1/2}$. The first one of these is related to the instability of the Couette flow. It was Landau [28] who originally suggested from general considerations that the amplitude of disturbances past marginal stability must increase like

$$A \sim (Re - Re_c)^{1/2} \qquad (81)$$

where Re is the Reynolds number characterizing flow beyond the onset of instability at Re_c. Stuart [29] extended the idea to rotating flows and explicitly showed from variational considerations that

$$A \sim C_T \sim (Ta - Ta_c)^{1/2}. \qquad (82)$$

With another extension, Chandrasekhar [30] obtained for buoyancy-driven flows

$$A \sim (Ra - Ra_c)^{1/2} \qquad (83)$$

Ra being the Rayleigh number (for related earlier work, refer to [31–33]). A similar argument may be

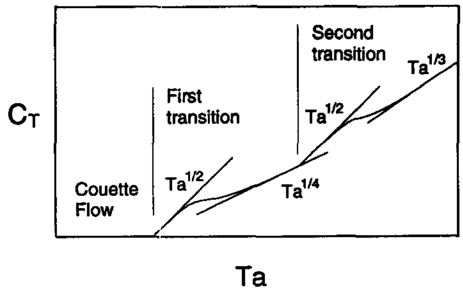

Fig. 2. Torque regimes.

conjectured for the second transition (Fig. 2). Further experimental studies are needed, however, before additional remarks on these transition regimes.

Studies on the heat transfer between concentric cylinders with an inner heated cylinder rotating and an outer cooled cylinder stationary have been confined to laminar flows. Bjorklund and Kays [10] propose a correlation

$$Nu = 0.175Ta^{1/4} \tag{84}$$

by assuming the analogy between heat and momentum transfer. A correlation including the effect of Prandtl number

$$Nu = \left(\frac{Ta}{Ta_c}\right)^{m/2}, \quad m = m(Pr) \tag{85}$$

is proposed by Ho *et al.* [12] who suggest $m = 0.07$ for their liquid metal data and $m = 1.31$ for the glycerol data of Hass and Nissan [11]. Also, a correlation obtained by Aoki *et al.* [13]

$$Nu = 0.22(Ta/F_g)^{1/4}Pr^{0.3} \tag{86}$$

fits the experimental data over the range of Ta from 5000 to 2×10^5 where F_g being geometric factor. When the gap is relatively large, Ball *et al.* [34] propose the following relationship:

$$Nu = 0.069\left(\frac{R_1}{R_2}\right)^{-2.9084} Ta^{0.2307ln(3.3361R_1/R_2)},$$

$$0.437 < \frac{R_1}{R_2} < 1. \tag{87}$$

In order to interpret the foregoing experimental data and unify the models proposed so far, consider equation (70) as an equality depending on two numerical constants, C_0 and C_1:

$$Nu = C_1\Pi_C^{1/4}, \quad \Pi_C = \frac{TaPr^2}{C_0+Pr}. \tag{88}$$

Equations (84)–(86) are rearranged in terms of

Table 1. $Nu/Ta^{1/4}$ vs Pr

Pr	$Nu/Ta^{1/4}$	Literature
0.025	0.0352	[12]
0.7	0.175	[10]
4.5	0.3454	[13]
45	0.6893	[13]
160	1.0084	[13]
750	1.0043	[11]

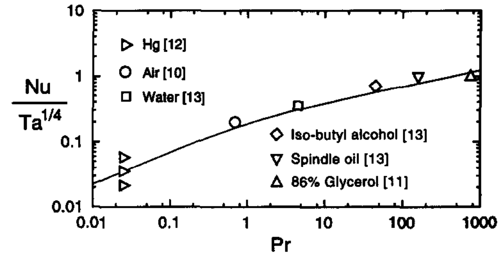

Fig. 3. Comparison between proposed correlation with $C_1 = 0.22$, $C_0 = 0.93$ and previous empirical correlations as $Nu/Ta^{1/4}$ vs Pr.

$Nu/Ta^{1/4}$ vs Pr as shown in Table 1. A correlation valid for any Prandtl number is proposed in terms of equation (88) as

$$Nu = 0.22\Pi_C^{1/4}, \quad \Pi_C = \frac{TaPr^2}{0.93+Pr}. \tag{89}$$

Figure 3 shows the dependence of this correlation on the Prandtl number and demonstrates the existence and fundamental significance of Π_C for rotating flows. Caution should be exercised on the numerical constant ($C_0 = 0.93$) of the denominator which is tentatively based on the average of the sparse mercury data. Additional data with other liquid metals (such as Bi, Na, K) are needed before a more reliable value can be predicted for this constant. The large Prandtl limit of equation (89)

$$\lim_{Pr\to\infty} Nu = 0.22(TaPr)^{1/4} \tag{90}$$

which is independent of C_0, agrees with the correlation proposed by Kim [35]. Figure 4 taken from Kim shows the correlation of the experimental data by equation (90).

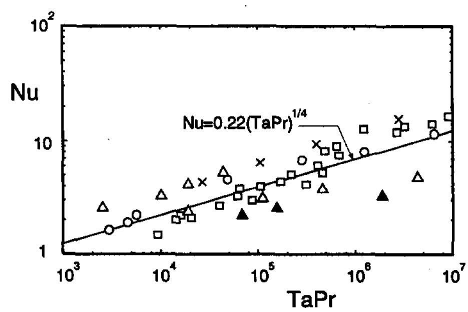

Fig. 4. Laminar heat transfer correlation valid for $Pr \geqslant 0.7$ (reproduced from [35]).

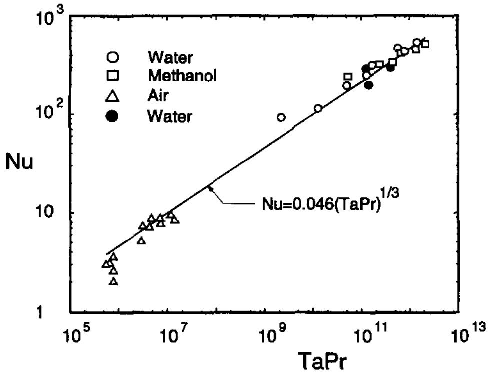

Fig. 5. Turbulent heat transfer correlation valid for $Pr \geqslant 0.7$ (reproduced from [36]).

Lack of turbulent experimental data for small Prandtl numbers precludes any correlation in terms of Π_C at this time. With the data available for $Pr \geqslant 0.7$, equation (76) becomes

$$Nu = 0.046(TaPr)^{1/3} \quad (91)$$

which is the correlation proposed by Tachibana [36]. Figure 5 taken from Tachibana shows the correlation of the experimental data by equation (91).

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